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Resistive G-Modes and Ballooning

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ABSTRACT

A unified theory of the linear stability of the Roberts and Taylor type of resistive interchange and ballooning is presented. The effects of both parallel and perpendicular viscosity as well as of finite shear and finite β are included in a MHD treatment of the problem. Kinetic effects are also studied. The "hybrid kinetic" model with Vlasov ions and guiding center electrons has been appropriately generalized to allow for electron-ion collisions. The geometry is that of a plane slab with magnetic shear. Toroidal curvature effects are simulated by the introduction of a gravitational acceleration $\hat{g}$, which varies along the magnetic field lines. Considering the limit of vanishing shear, we distinguish three different types of modes in the high perpendicular wave number limit as the magnitude of $\hat{g}$ is varied with respect to α/L^2; here α is a free parameter that simulates the average curvature and L is the connection length of the system. Finite viscosity and ion kinetic effects on these modes are being considered. In the case of a configuration with finite shear an expansion of the pertinent quantities in terms of Hermite functions enables us to derive a characteristic secular determinant for the problem. In the limit of large L the resistive G-mode is obtained subject to viscous corrections. As L becomes finite the ballooning mode results. Finally, a numerical scheme has been devised to

solve the equations when all of the various effects of interest are simultaneously present.

I. Model: Using resistive MHD theory with an anisotropic pressure tensor the resistive interchange with parallel wavenumber $k_\parallel \sim 0$ and its special form, the ballooning mode, with $k_\parallel \sim 1/L$ are derived as limits of a single model. Advantage is taken of the simplicity of the magnetized plane slab geometry to simultaneously study the effects of both parallel and perpendicular viscosity, finite shear and finite beta. Magnetic curvature effects are simulated by a gravitational acceleration[1]

$$G = G_o(-\alpha + \cos\frac{2\pi}{L} z), \quad G_o \equiv \frac{v_{th}^2 a}{v_r v_h R_c} = \beta_o S \frac{a}{R_c} \tag{1}$$

where x, y, z are Cartesian coordinates, L is the connection length along a magnetic field line, v_{th}, v_r, v_h are the thermal, resistive, and Alfvén speeds defined with respect to a characteristic length a. R_c is the simulated radius of curvature. $S = v_h/v_r$, $\beta_o = 2p_o/B_o^2$, p_o, B_o are characteristic values of the kinetic pressure and of the strength of the confining field respectively. α is a constant representing average curvature.

We write for the equilibrium magnetic field

$$\underset{\sim}{B}^{(o)} = (0, B_y^{(o)}(x), B_z^{(o)}(x)) \tag{2}$$

and we do not make any a priori assumption about the relative magnitude of its two components.

For the same simple geometry the effect of finite Larmor radius on resistive interchanges is also studied. For this purpose the "hybrid kinetic" model with Vlasov ions and guiding center electrons[2] is appropriately generalized to include electron-ion collisions.

II. Resistive MHD: We employ Maxwell's equations along with the usual resistive MHD set:

$$\rho\frac{d\underline{v}}{dt} = S(-\beta_0\nabla p + \underline{J}\times\underline{B}) + 2pG\hat{e}_x - U\nabla\cdot\underline{\underline{\Pi}} \tag{3}$$

$$S\frac{\partial p}{\partial t} + \underline{v}\cdot\nabla p + \gamma p\nabla\cdot\underline{v} = 0, \quad \underline{E} + \underline{v}\times\underline{B} = \underline{J} \tag{4}$$

where

$$\underline{\underline{\Pi}} \equiv -3(\hat{\underline{b}}\hat{\underline{b}} - \frac{1}{3}\underline{\underline{I}})[\hat{\underline{b}}\cdot\nabla\underline{v}\cdot\hat{\underline{b}} - (\hat{\underline{b}}\cdot\nabla\hat{\underline{b}})\cdot\underline{v} - \frac{1}{3}\nabla\cdot\underline{v}] + U_\perp[\nabla\underline{v} + (\nabla\underline{v})^T - \frac{2}{3}\underline{\underline{I}}\nabla\cdot\underline{v}] \tag{5}$$

$\underline{\underline{\Pi}}$ is the anisotropic pressure tensor.[3] We have normalized velocity to v_r, time to a/v_h, p to $2p_0$, $\underline{E}$ to $v_r B_0$, ρ to ρ_0, $\underline{B}$ to B_0, and $\underline{x}$ to a. We let $\hat{\underline{b}} = \underline{B}^{(0)}/|\underline{B}^{(0)}|$ and

$$U = v_v/v_h = \frac{3}{2}\beta_0 S\,\Omega^{-1}, \quad \Omega = \frac{3}{2}\beta_0\, v_{th}^2/v_r v_v, \quad U_\perp = \mu_\perp/\mu_\parallel, \quad v_v = \mu_\parallel/a\rho_0$$

where $\mu_\parallel$ and $\mu_\perp$ are the parallel and perpendicular coefficients of viscosity respectively. We consider "quasi-modes" of the form[1]

$$f^{(1)} = f(\delta x,z)\,\exp\left[iK(B_y^{(o)}z - B_z^{(o)}y) + \omega t\right] \tag{6}$$

with $\delta<<1$, $K>>1$. Using Eq. (6), operating with $\hat{e}_x\cdot\nabla\times\nabla\times$ on Eq. (3) we derive the equation of motion perpendicular to the line of force

$$\frac{\partial}{\partial z}\,\frac{1+\hat{s}^2z^2}{1+\hat{s}^2z^2+\hat{\omega}/\hat{K}^2B_z^{(o)2}}\,\frac{\partial}{\partial z}\,v_x - \hat{\omega}\,\frac{\hat{K}^2\rho^{(o)}}{S^2}\,(1+\hat{s}^2z^2)v_x$$

$$-\frac{M_\perp}{S}\,\hat{K}^4\,B_z^{(o)2}\,(1+\hat{s}^2z^2)^2\,v_x + \frac{2\hat{K}^2}{S}\,\frac{\hat{\Gamma}}{\hat{\Omega}}\,\gamma p^{(o)}G(z)\hat{p} = 0 \tag{7}$$

Here $\hat{s}(x) \equiv \left(B_z^{(o)}\,B'^{(o)}_y - B'^{(o)}_z B_y^{(o)}\right)/\left(B_z^{(o)}\,B^{(o)}\right)$ is the shear parameter. In addition, we have defined

$$\hat{\Omega} = \hat{Q} + \hat{\omega},\quad \hat{Q} \equiv \gamma p^{(o)}Q,\quad \hat{\omega} \equiv \omega S,\quad \hat{K} = KB^{(o)}/B_z^{(o)}$$

$$M_\perp \equiv \Pi_\perp,\quad \hat{\Gamma} \equiv -p'^{(o)}/\gamma p^{(o)} - \frac{3}{2}\mathcal{X},\quad \mathcal{X} \equiv V_\perp v_\perp/v_x,\ \hat{p} = \frac{\hat{\Omega}}{\gamma p^{(o)}\hat{\Gamma}}\,p^{(1)}$$

Parallel viscosity is neglected in Eq. (7) as it can only introduce oscillation. However, this quantity plays an important role in the equation describing parallel motion

$$\frac{3}{2}\left(1+\frac{B_y^{(o)2}}{B_z^{(o)2}}\right)\frac{p^{(o)}\hat{\omega}}{\Pi\hat{\Gamma}S}\left[(\hat{\Gamma}+5/2\,\mathcal{X})v_x - \frac{\hat{\omega}\hat{\Gamma}}{\hat{\Omega}}\hat{p}\right] = -\frac{\partial^2}{\partial z^2}\hat{p} + \frac{\partial^2}{\partial z^2}v_x\ . \tag{8}$$

We distinguish the following cases of interest:

A. <u>Stationary Modes; $\omega=0$</u>: A critical pressure gradient is obtained for the onset of ballooning ($\mu_\perp = 0$):

$$\hat{\Gamma}\beta_0|_c \simeq (Q/K^2) B_y^{(o)2}/(B_y^{(o)2} + B_z^{(o)2}) \quad .$$

Parallel viscosity is seen to be responsible for the existence of a threshold which is not possible when resistivity is the only source of dissipation. The shape of the mode at marginality has also been obtained and it would be useful in the context of a quasilinear transport theory.

B. <u>Growing Modes</u>; $\omega \neq 0$:

a. <u>No Shear; $\hat{s}=0$</u>: This is appropriate for devices like Field Reversed Theta Pinches and multipoles particularly in view of the fact that our treatment is valid for finite beta. We define

$$\hat{q} \equiv 4K^2\hat{g}(\hat{K}^2 B_z^{(o)2} + \hat{\omega})/\hat{\Omega}, \quad \hat{g} = G_0\hat{\Gamma}\gamma p^{(o)}, \quad \hat{R} = L_\parallel \pi B_z^{(o)}$$

and examine the limits:

1. $\hat{q} \ll 1$: For $\hat{R}^2\hat{g} < \alpha$ we find a viscoresistive ballooning root

$$\omega = S^{-1}\hat{g}\hat{R}^2 \left(\hat{K}^2 B_z^{(o)2} - \hat{Q}\right)/(\alpha - \hat{R}^2\hat{g}) - S^{-1}\hat{Q} + O(S^{-2}) \quad . \tag{9}$$

For $\hat{R}^2\hat{g} = \alpha$, a "resonant" mode

$$\omega = 2^{1/2}\hat{g}\hat{R}S^{-1/2}\left(\hat{K}^2 B_z^{(o)2} - \hat{Q}\right)^{1/2}/\left(M_1 B_z^{(o)2}\hat{K}^2\right)^{1/2} - \hat{Q}S^{-1} + O(S^{-2}) \tag{10}$$

and for $\hat{K}^2\hat{g} > \alpha$ an ideal interchange mode being acted on by dissipation

$$\omega = 2\hat{g}(\hat{R}^2\hat{g} - \alpha)/(M_\perp B_z^{(o)^2}\hat{K}^2) + O(S^{-1}), \tag{11}$$

(for $M_\perp \hat{K}^2 \gg 0$).

2. $\hat{q} \gg 1$.: For this case, the dispersion relation becomes a quintic in $\hat{\omega}$. Simple formulae are not very accurate. Using MACSYMA only two of the roots are found to be growing. Large perpendicular viscosity combined with small parallel viscosity can stabilize a resistive ballooning-like root for high and an ideal ballooning-like root for intermediate K.

In general $\mu_\parallel$ is found to be effective in stabilizing these modes for very small values of $\hat{g}$, viz. Sec. 2, however, for larger $\hat{g}$ it is destabilizing. $\mu_\perp$ is always stabilizing and becomes most effective as $K \to \infty$.

b. Finite Shear; $\hat{s} \neq 0$: This case is important for shear stabilized devices (Reversed Field Pinch), but also for tokamaks with her peaked current profiles. We obtain simple analytic results using the "disconnected mode approximation"[5] and we make use of an expansion in terms of Hermite functions to treat the general problem.

1. Neglecting parallel inertia and electromagnetic effects we find (for $\mu_\parallel = 0$)

$$\tilde{\omega} = 1 - \frac{(1+\tilde{Q})}{(3+\tilde{Q})}\Lambda + \ldots \text{ for } \Lambda \ll 1,$$

$$\tilde{\Omega} = \Lambda^{-1} + \frac{\tilde{Q}}{(1+\tilde{Q})^2}\Lambda^{-1} - \ldots \text{ for } \Lambda \gg 1 \tag{12}$$

where the tilde denotes normalization to ω_g, the growth rate for the "twisted slicing mode"[3] ($\mu_\parallel \neq 0$).

$$\Lambda \equiv \hat{s}\omega_g S/\left(4\hat{\rho}(1-\alpha)^2\hat{K}^2\hat{R}^2 B_z^{(o)2}\right)$$

The first of Eq. (12) represents the G-mode limit, while the second is the ballooning limit.

2. Letting $\mu_\perp \neq 0$ and using the same approximation as above as well as an expansion in terms of Hermite functions, we derive a secular infinite matrix determinant for the problem. Using just the first term of this determinant we find very roughly (as $L \to \infty$)

$$\omega = -\hat{\Omega} + \frac{\omega_g + \hat{\Omega}}{KB^{(o)}}\left(\frac{\rho^{(o)}\omega_g}{2M_\perp}\right)^{1/2},$$

predicting a cutoff K. Letting, for instance, $M_\perp \sim 10^{-3}$, $S \sim 10^6$ we find $K_{cutoff} \sim 20\ (\hat{g}/\hat{s})^{1/2}$.

3. Utilizing the expansion in terms of Hermite functions, the general system of Eqs. (8) and (9) is solved also. Successive disconnected mode approximations are performed for calculating the eigenfunctions. The particular iterative scheme involved displays fast convergence. Growth rates are found to be greatly suppressed with respect to the $\hat{s} = 0$ case.

III. The Hybrid Kinetic Model

Current literature on resistive G-modes and ballooning does not address the effects of finite ion Larmor radius (as compared to relevant gradient scale lengths) beyond lowest order. A fully ion-kinetic treatment is necessary in order to take into account the finiteness of the ion orbits and the electrostatic nature of ion confinement. Such a treatment is important to high temperature and high S regimes, which would be obtainable near or at reactor conditions. Here we let r_{Li} denote the thermal ion Larmor radius and we assume $kr_{Li} \sim O(1)$. We consider collisionless ions in static equilibrium. They are characterized by an equilibrium distribution $f_i^{(o)}$, which only depends on the ion Hamiltonian H. It includes a gravitational potential according to Eq. (1). Resistivity is assumed to be entirely due to collisions between electrons and ions. Resistive effects manifest themselves only through the generalized Ohm's law. On the other hand gravity only affects the momentum balance equation. We note that for our treatment to be correct we must have

$$r_{Li}|\cdot|/v_r >> 1 \quad . \tag{13}$$

Given Eq. (13), we can generalize the recently developed hybrid-kinetic formalism of Ref. (2) to include constant resistivity and gravitation. We linearize the ion and electron kinetic equations assuming magnetically confined electrons ($|\omega| << \omega_{ce}$, $r_{Le} << a$) but electrostatically confined ions ($|\Omega| \lesssim \omega_{ci}$, r_{Li}/a finite). Here ω_c denotes cyclotron frequencies and the subscripts i, e refer to the ions and electrons respectively. The confining electric field is given by

$$\underset{\sim}{E}^{(o)} = \frac{\nabla p_i^{(o)}}{eN} - \frac{m_i}{e}\underset{\sim}{g} \tag{14}$$

where N is the unperturbed number density, m_i the ion mass, $-e$ the electronic charge and $\underset{\sim}{g}$ the gravitational acceleration. The detailed collisional processes needed in the presence of finite electron pressure are approximated by a BGK model.[6]

We base our analysis on the following equations.

a. The ion Vlasov equation, unexpanded.

b. Guiding center electrons with a BGK collision term.

c. The parallel component of Ohm's law along with Faraday's law.

We stress, however, that the present formalism is quite insensitive to the specific electron model, which can equally well be chosen to be that of a massless, finite pressure resistive fluid.

Linearizing, we obtain a system of coupled eigenvalue equations from momentum balance

$$\nabla_\perp \times \left[(\nabla\times\mathbf{B}^{(1)}) \times \mathbf{B}^{(0)} + (\nabla\times\mathbf{B}^{(0)}) \times \mathbf{B}^{(1)} \right] = \nabla_\perp\times \boldsymbol{\mathcal{F}}^{(1)} + \nabla_\perp\times \boldsymbol{\Pi}^{(1)} \tag{15}$$

where $\boldsymbol{\mathcal{F}}^{(1)} \equiv -\omega e \int d^3v (\mathbf{E}^{(0)} + \mathbf{v}\times\mathbf{B}^{(0)}) \dfrac{\partial f_i^{(0)}}{\partial H}$

is the generalized inertial force density and

$$\boldsymbol{\Pi}^{(1)} = m_i \mathbf{g}(p_{e\parallel} + \xi_\perp\cdot\nabla p_e^{(0)}) N^{-1} \int d^3v \frac{\partial f_i^{(0)}}{\partial H} \mathcal{L}$$

is a known function of $(\mathbf{I} - \hat{\mathbf{b}}^{(0)} \hat{\mathbf{b}}^{(0)})\cdot\hat{\mathbf{j}}$. From Ohm's law we have

$$(\omega - \mathbf{v}_0^*\cdot\nabla \ln B^{(0)} - \mathbf{v}_0^*\cdot\nabla)\, \xi_\parallel B^{(0)} = \eta_\parallel \left(\mathbf{A}^{(1)}\cdot\nabla\times\mathbf{J}^{(0)}/B^{(0)} + \hat{\mathbf{b}}^{(0)}\cdot\nabla\times\mathbf{B}^{(1)}\right) \tag{16}$$

Here we have used the gauge

$$\phi^{(1)} = p_{e\parallel}/eN + \mathbf{v}_0^*\cdot\mathbf{A}^{(1)} \quad , \tag{17}$$

assuming $\mathrm{Re}(\omega) \neq 0$, and we have defined the mathematical variable ξ via

$$\underset{\sim}{B}^{(1)} \equiv \nabla\times\left(\underset{\sim}{\xi}_\perp\times\underset{\sim}{B}^{(o)} + \xi_\parallel\underset{\sim}{B}^{(o)}\right) \equiv \nabla\times\underset{\sim}{A}^{(o)} \quad . \tag{18}$$

In addition, we employ

$$\frac{d\mathcal{Y}}{dt} = m_i\underset{\sim}{v}\cdot\frac{d}{dt}\,\xi_\perp - \left(p_{e\parallel}^{(1)} + \xi_\perp\cdot\nabla p_e^{(o)}\right) + m_i\xi_\perp\cdot\underset{\sim}{g} + e\,\underset{\sim}{v}\cdot\underset{\sim}{B}^{(o)}\xi_\parallel \quad . \tag{19}$$

$\mathcal{Y}$ is seen to be an integral over the equilibrium ion orbits

$$\frac{d\underset{\sim}{v}}{dt} = \frac{e}{m_i}\left(\underset{\sim}{E}^{(o)} + \underset{\sim}{v}\times\underset{\sim}{B}^{(o)}\right) + \underset{\sim}{g} \quad .$$

$\phi^{(1)}$ is the perturbed electrostatic potential.

In deriving Eqs. (15) and (16), no assumptions were made concerning magnetic shear but terms of $O(g^2, \eta g)$ have been dropped.

As a simple example we consider the case of small shear, $B_y^{(o)}/B^{(o)} \sim \varepsilon$, and negligible temperature ratio T_e/T_i. Here $\varepsilon^4 = r_{Li}{}^4|\partial/\partial x \;\; \ln|\xi||^4 \ll 1$. Then to leading order in ε^2 we have[7]

$$\nabla_\perp \times \left[\omega^2\rho\xi_\perp + \omega\rho(1+\tfrac{1}{2}\beta)\underset{\sim}{V}_E\cdot\nabla\xi_\perp\right] =$$

$$\nabla_\perp \times \left[\nabla\times\underset{\sim}{B}^{(1)}\times\underset{\sim}{B}^{(o)} + (\nabla\times\underset{\sim}{B}^{(o)})\times\underset{\sim}{B}^{(1)}\right] \tag{20}$$

$$- \nabla_\perp \times \left[\underset{\sim}{g}\,\xi_\perp\cdot\nabla\rho - \omega m_i\underset{\sim}{g}\int d^3v\,\frac{\partial f_i^{(o)}}{\partial H}\,\mathcal{Y}_o\right]$$

$$\underset{\sim}{B}^{(1)} = \nabla\times(\xi_\perp\times\underset{\sim}{B}^{(o)}) + \frac{\eta}{\omega+i\underset{\sim}{k}\cdot\underset{\sim}{V}_e}\nabla^2\underset{\sim}{B}^{(1)} \tag{21}$$

where we assumed $Ka >> 1$. $\mathcal{S}_o$ is a resonant integral[7] only depending on $\xi_\perp$ for $|\omega|a/v_r >> 1$, and $\underset{\sim}{V}_E$ is the cross-field drift velocity.

It is seen from Eq. (20) that a leading order resonance occurs due to the effect of gravity. If we arbitrarily neglect it then Eqs. (20) and (21) can be solved using standard methods of resistive MHD.

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